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Reliability-Based Structural Optimization: A Proposed Analytical-Experimental Study

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A Proposed Analytical-Experimental Study

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Summary

An analytical and experimental study for assessing the potential of reliability-based structural optimization is proposed and described. In the study, competing designs obtained by deterministic and reliability-based optimization are compared. The experimental portion of the study is practical because the structure selected is a modular, actively and passively controlled truss that consists of many identical members, and because the competing designs are compared in terms of their dynamic performance and are not destroyed if failure occurs. The analytical portion of this study is illustrated on a 10-bar truss example. In the illustrative example, it is shown that reliability-based optimization can yield a design that is superior to an alternative design obtained by deterministic optimization. These analytical results provide motivation for the proposed study, which is underway.

Introduction

Over the last two decades significant advancements have taken place in both the theory of and computational techniques used in structural reliability (Madsen et al. [86], Melchers [87], and Cruse [92]). Studies have demonstrated analytically that reliability-based optimization can be more effective than classical deterministic optimization for designing aircraft structures (e.g., Yang, Nikolaidis, and Haftka [90]).

However, the superiority of probabilistic over classical deterministic methods has not been proven in real life design or by experiment. This is one of the reasons that reliability-based optimization has not been applied to aerospace structures. (In this paper, the terms "reliability-based optimization" and "probabilistic optimization" are used interchangeably to refer to optimization in which uncertainties and probability of failure are taken into account.)

Three critical issues involving the accuracy of and basic assumptions behind the methods for structural reliability assessment are:

- In reliability-based design, it is difficult to quantify all important uncertainties. For example, little is known of how to quantify uncertainties that are due to assumptions and simplifications in analysis procedures (modeling uncertainties).

- In many cases, there are significant errors in the assumed probability distributions of some random variables, such as, for example, the loads. Usually, the probability distribution of a load is estimated by analyzing sample values that are in the vicinity

of the mean value and not in the right tail of the probability distribution. However, since most failure cases occur when a load is large, it is the right tail of the probability distribution of a load that is important in reliability assessment. Similarly, it is difficult to determine the shape of the left tail of the probability distribution of material properties, which is also critical in reliability assessment. For some random variables, including the load and material properties just cited, a small error in the assumed distribution may cause a large error in the estimated probability of failure. This problem, which is often referred to as the *tail sensitivity problem*, is a serious consideration when assessing reliability (Melchers [87], Ben-Haim and Elishakoff [90]).

- As a result of the aforementioned problems, the probability of failure that is calculated in reliability analysis, and used in reliability-based design, can be significantly different from the actual failure probability. It is generally agreed that this "nominal" failure probability should be interpreted as a subjective measure of safety rather than as the actual failure probability. Consequently, it is important to answer the following question. If this nominal failure probability is used as a surrogate for the actual failure probability in reliability-based optimization, will the resulting designs still be better than their deterministic counterparts?

If designers are to accept probabilistic methods as practical design tools, it is important to demonstrate experimentally that probabilistic methods can yield better designs than deterministic methods, despite the above difficulties. However, demonstrating the advantages of reliability-based optimization is not a simple task. To do so, many pairs of alternative structures obtained using reliability-based optimization and deterministic optimization must be constructed, tested, and their performance compared based on specified failure criteria. Depending upon the failure criteria selected, it might be necessary to destroy a large number of sample structures to obtain a valid assessment. Since destroying a large number of structures would be impractical for this study, the structure and its failure modes were chosen in a way that failure would not imply its destruction.

The objectives of this report are to:

- Propose and describe an analytical-experimental study to assess the advantages of reliability-based optimization. The study will establish a practical procedure to compare reliability-based optimization with deterministic optimization.

- Demonstrate with an analytical example that reliability-based optimization can lead to a design that is superior to a design obtained by deterministic optimization. Use the example to illustrate the analytical part of the proposed study.

The key feature of the study is to use an actively and passively controlled modular truss structure and consider a large number of failure events which do not imply the destruction of the structure or its members. Such a failure event occurs, for example, when the damping ratio of a vibration mode falls below a specified value and/or when the vibration amplitude at some given location exceeds a maximum allowable value. During tests, the

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truss can be disassembled and reassembled after rearranging its members randomly. Thus, a large number of identical random samples of the same design can be tested at low cost. The large number of random samples, together with the large number of failure events, eliminates the factor of chance when comparing a probabilistic design with an alternative deterministic design, and makes it possible to draw valid conclusions regarding the effectiveness of reliability-based optimization.

In the proposed study, two trusses are designed – one using deterministic optimization and the other using reliability-based optimization. In both cases the objective is to maximize safety; however, because the definition of safety differs in the two cases, the final designs are different. Then, a large number of both designs are constructed and tested. The design that has the smaller number of failures is accepted as being better.

In this paper, the general procedure for comparing deterministic optimization with reliability-based optimization is presented first. Then, the structure that is proposed for the study is described. Finally, an analytical example involving a ten-bar truss is used to provide motivation for the study and to illustrate some of its steps. When compared, the probabilistic design that is obtained in the example is superior to the deterministic design. The methodology for assessing the reliability is presented in an appendix.

Symbols

c	total cost of damping control system
c_0	maximum allowable total cost of damping control system
F_i	failure mode associated with the ith vibratory mode
F_s	system failure
$P(F_s)$	probability of system failure
R_s	system reliability, $1 - P(F_s)$
x_i	design variables. For the deterministic design, x_i are the normalized gains of active members 1 and 2 and the normalized damping factors of passive members 3 and 4. For the probabilistic design, x_i are the mean values of the normalized gains of active members 1 and 2 and the mean values of the normalized damping factors of passive members 3 and 4.
β_s	system reliability index (or, system safety index)
ζ_i	damping factor of the ith vibratory mode
$\zeta_{0,i}$	lowest acceptable damping factor for the ith vibratory mode
$\Phi(.)$	probability distribution function of a standard Gaussian random variable

Description of Analytical-Experimental Study

The proposed study consists of an analytical part and an experimental part. In the analytical part, two trusses are designed – one using reliability-based optimization and the other using deterministic optimization. In each case, the objective is to maximize safety. Safety is quantified by the margin of safety in deterministic optimization and by the system reliability in reliability-based optimization. After the two trusses are designed, their system reliabilities are evaluated and compared analytically.

In the experimental part, many samples of the probabilistic and deterministic designs are tested to determine their dynamic performance. Then, the number of failure events for the two types of designs are compared. Failure events are defined by unacceptable dynamic behavior. Examples include cases where the damping ratio of any of the modes is smaller than a prescribed minimum value, or the vibratory amplitude of any node is larger than a prescribed maximum value. In the tests, if the percentage of failures for the probabilistic designs is smaller than the percentage of failures for the deterministic designs, it is concluded that the probabilistic designs are safer and that reliability-based design is an effective approach for structural design.

Analytical Part of Study, Probabilistic and Deterministic Design

When a structure is designed **probabilistically** to maximize its safety, the objective is to minimize the probability of system failure, $P(F_s)$. This is done subject to the requirement that the utilized resources (cost, weight, energy expended) do not exceed the allocated resources. It is assumed that the system fails if its dynamic behavior is unacceptable.

For example, failure of the system can be defined as the event in which the damping ratio of any vibratory mode falls below a lowest acceptable value for that mode. Specifically, the failure mode associated with the ith vibratory mode, F_i, is an event that occurs if the damping factor of the ith vibratory mode, ζ_i, becomes less than the lowest acceptable damping for that mode, $\zeta_{0,i}$[1]. That is,

$$F_i: \quad \zeta_i < \zeta_{0,i} \qquad i = 1,...,n \tag{1}$$

When a structure is designed **deterministically** to maximize its safety, the objective is to maximize the margin against failure. Using the above example and notation, the margin of safety for failure mode i is defined to be $\zeta_i - \zeta_{0,i}$. Thus, the objective is

$$\text{Maximize:} \quad min \ (\zeta_i - \zeta_{0,i}) \qquad i = 1,...,n \tag{2}$$

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[1] Because of this definition, there is a one-to-one correspondence between failure modes and vibratory modes. Therefore, the same subscript is used to specify the failure mode and vibratory mode number. In general, there is no one-to-one correspondence between failure modes and vibratory modes.

such that the utilized resources do not exceed the allocated resources. The same resources are allocated in deterministic and probabilistic design.

After the two alternate trusses are designed, their probabilities of failure are evaluated analytically and compared. If the probabilistic design has a significantly lower failure probability than the deterministic design (for example if it is 50% of the failure probability of the deterministic design), then it is likely that the probabilistic design will perform better than the deterministic design in an experiment in which many samples of the two designs are tested. The experimental part of the study is undertaken if, and only if, the failure probability of the probabilistic design is significantly lower than that of the deterministic design. Otherwise, the design requirements are redefined (by changing the required damping factor, allowable vibration amplitude, etc.) to produce two new designs that do have significantly different failure probabilities. These new designs can be used in the experimental part of the study.

Experimental Part of Study

Testing involves a large number of pairs of structures, each pair consisting of a design obtained by deterministic optimization and a design obtained by reliability-based optimization. The number of structures of each type that fail is recorded. If the mathematical models that describe the uncertainties and structural response are sufficiently accurate, then it is likely that more deterministic than probabilistic designs will fail. This will demonstrate that the probabilistic design is more reliable than its deterministic counterpart because, while the same resources are used in designing both structures, the probabilistic design is less likely to fail.

Description of Experimental Structure

A modular truss was selected for the study because, as explained in the introduction, it is suitable for the experimental procedure. The truss consists of two sets of identical struts bolted together using joints. Concentrated masses can be attached to the joints. The dynamic behavior of the truss is controlled by using active struts and/or passive dampers. A typical truss (without dampers) is shown in Figure 1.

Active struts consist of piezoelectric sensors and actuators with integral control. This type of active member is described by Preumont et al. [91], and Ponslet et al. [91].

Passive dampers can be constructed by coating struts with viscoelastic material, which allows the members to absorb energy. The behavior of the passive members is nonlinear because both the damping factor and the stiffness depend on the displacement amplitude. Moreover, the damping factor depends on the temperature and the frequency. Jones [80] reviewed the characteristics of viscoelastic materials when used for damping applications. Other passive damping concepts will also be explored.

There is uncertainty in the damping ratio and stiffness of both the passive and active members, which is due to sample-to-sample variability and the dependence of these quantities on both the deformation and temperature of these members. In addition, the dimensions of the structural members, the material properties, and the weights of the concentrated masses vary randomly due to manufacturing imperfections.
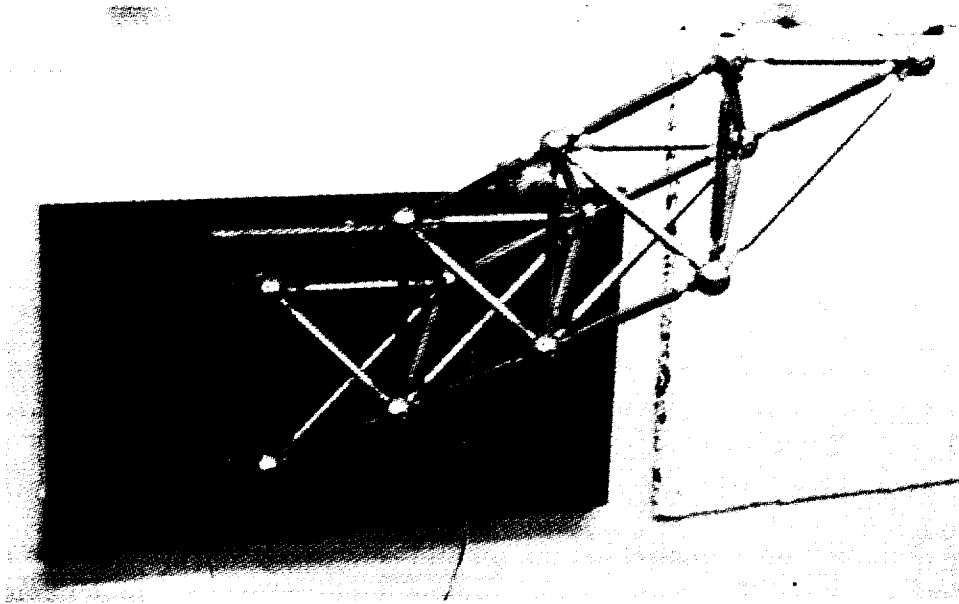
Figure 1.- Typical experimental truss, without dampers. (From Ponslet et al. [93])

▶ Active members   ⊐ Passive members   ● Joints with concentrated masses
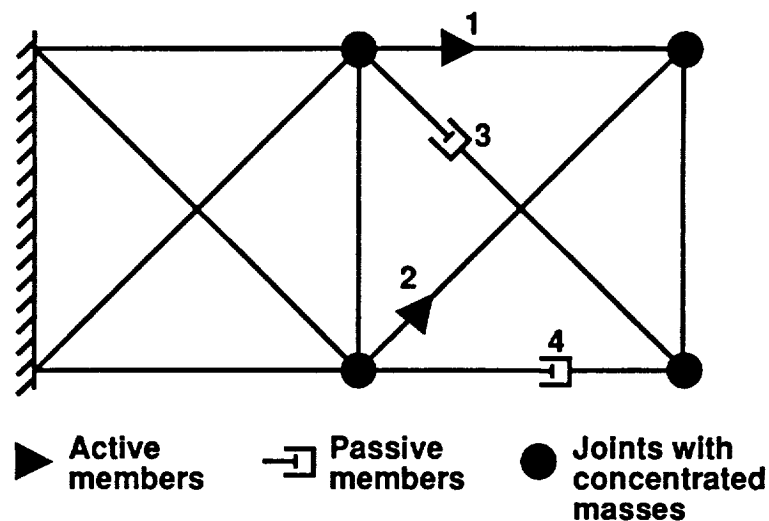
Figure 2.- Ten-bar truss example. Numbers indicate locations of the corresponding design variables.

Example: Ten-Bar Truss

The following analytical example illustrates the procedure described above and demonstrates that reliability-based optimization can produce a superior design. In the example, two ten-bar trusses are designed and compared. One truss is designed using deterministic optimization and the other using reliability-based optimization. In both cases the objective is to maximize safety. Topics presented include the optimization procedures, the resulting two designs, and the performance of the two designs.

Each of the two alternative designs was assumed to fail when the damping ratio of any of the first four natural vibration modes fell below a specified value. Two active and two passive members were used to control damping. The truss configuration and the locations of active and passive members and concentrated masses are shown in Figure 2. Each active member consisted of a piezoelectric stack to provide actuation and a force transducer to provide sensing. An integral control law was used. The truss properties that were used in the optimization are presented in Table 1.

Table 1. Properties of Ten-Bar Truss and Design Requirements

Cross-sectional area of elements	0.132×10^{-3} m^2
Length of short members	9. m
Young's modulus	$70. \times 10^9$ N/m^2
Density	$3. \times 10^3$ kg/m^3
Concentrated masses	10. kg
Nominal gains of active members	0.5×10^{-4} m/Nsec
Nominal damping factors of passive members	200. Nsec/m
Lowest acceptable damping ratio	2.5%
Maximum allowable cost	$249.
Cost of active members	$100. /unit of normalized gain
Cost of passive members	$20. /unit of normalized damping factor

Note: Values selected for the unit costs are arbitrary and are used for illustrative purposes only.

Optimization Procedures and Results

In this section, the deterministic and probabilistic optimization procedures are summarized and the final designs are presented. The two optimization procedures differ only in the objective function. In the deterministic case, the objective is to maximize the minimum margin of safety. In the probabilistic case, the objective is to minimize the probability of system failure. In both cases, the algorithm that was used for the optimization was an extended interior penalty function technique incorporated in the code NEWSUMT-A (Grandhi et al. [85]).

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Deterministic Optimization

The optimization procedure consisted of maximizing the lowest margin of safety. Failure was defined as a damping ratio that was less than a specified allowable value for the first four vibration modes. Because the allowable value was the same (2.5%) for each vibration mode, the optimization procedure reduces to maximizing the lowest damping ratio. During the optimization, the total cost associated with the active and passive members was not allowed to exceed the maximum allowable cost. The optimization is defined formally as

$$\text{Maximize:} \quad min \ (\zeta_1,...,\zeta_4)$$

$$\text{Subject to:} \quad c = 100(x_1 + x_2) + 20(x_3 + x_4) \leq c_0 \qquad (3)$$

where ζ_i is the damping ratio in the ith mode, c is the total cost, and c_0 is the maximum allowable cost. Design variables were the normalized gains, x_1 and x_2, of the active members and the normalized damping factors, x_3 and x_4, of the passive dampers. The subscript on x indicates the member number. Locations of the active and passive members are shown in Figure 2.

Table 2 presents the values of the gains and the damping factors together with the costs of each active and passive member for the deterministic optimum.

Table 2. Deterministic Optimum for Ten-Bar Truss Example

Member type / Member number	Normalized gain or damping factor	Cost ($)
Active/1	1.848	184.8
Active/2	0.530	53.0
Passive/3	0.559	11.18
Passive/4	0.0	0.0

Reliability-Based Optimization

Reliability-based optimization accounts for uncertainties. There are many types of uncertainties, but, for this example, only the uncertainties associated with the active and passive members were considered; it was assumed that they were the most important. These uncertainties can be classified into two categories: modeling and random. Modeling uncertainties are due to simplifications in modeling and analyzing the structure. Random uncertainties are due to:

- variability between samples,

- variability in the conditions under which the members operate (temperature, amplitude of vibration) during the same experiment or from one experiment to another.

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Only random uncertainties in the gains and the damping factors were considered. The gains and the damping factors were assumed to be independent, Gaussian random variables with coefficients of variation equal to 10%.

After the uncertainties were defined, the truss was optimized by minimizing the probability of system failure, $P(F_s)$. As in the deterministic design, the total cost associated with the active and passive members was not allowed to exceed the maximum allowable cost. Formally, the following optimization problem was solved :

$$\text{Minimize:} \quad P(F_s)$$
$$\text{Subject to:} \quad c = 100(x_1 + x_2) + 20(x_3 + x_4) \leq c_0 \qquad (4)$$

Calculating $P(F_s)$ can be a formidable task. Fortunately, there are approximate procedures for making that calculation that are, in most cases, reasonably accurate. Examples include the first- and second-order Ditlevsen bounds (Madsen et al. [86], Melchers [87]). In the present study, the second-order, upper Ditlevsen bound was used to estimate the system failure probability. A brief explanation of the method is presented in the appendix.

The maximum allowable cost, c_0, was the same as that used in the deterministic optimization. Design variables were the mean values of the normalized gains, x_1 and x_2, of the active members (members 1 and 2) and the mean value of the normalized damping factor, x_3, of one of the passive dampers (member 3). The damping factor, x_4, of the other passive damper (member 4) was assumed to be zero for the following two reasons. First, in the deterministic design x_4 was found to be zero. Second, as is shown in the following section, the system reliability index for the deterministic design was found to be insensitive to x_4.

Table 3 presents the mean values of the gains and the damping factors together with the costs of each active and passive member for the reliability-based optimum.

Table 3. Reliability-Based Optimum for Ten-bar Truss Example

Member type/ Member number	Mean values of normalized gain or damping factor	Cost ($)
Active/1	1.941	194.1
Active/2	0.413	41.3
Passive/3	0.675	13.5
Passive/4	0.0	0.0

Description and Comparison of Deterministic and Probabilistic Designs

Deterministic Design

The design space for the deterministic optimum is shown in Figure 3. Recall that the objective was to maximize the minimum ζ_i subject to cost ≤ $249, and that the problem was solved using mathematical programming techniques. Cost and damping ratios increase toward the upper right. The optimum design is bounded from above by the line defined by cost=$249. A move to the left along that line causes ζ_4 to be reduced; a move to the right causes ζ_1 and ζ_2 to be reduced. At the optimum, the damping ratio of the third vibration mode is 3.1%; for the first, second, and fourth modes, the damping ratios are all 3.0%. Note that, according to the deterministic analysis, mode 3 is less important than the other modes because its damping ratio is highest.
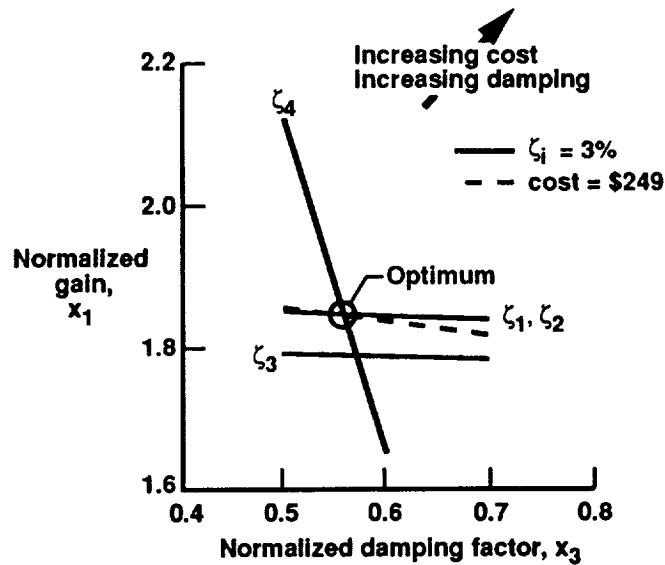
Figure 3.- Design space for deterministic optimum. Each curve, except cost, defines a 3% damping ratio for the corresponding vibration mode. $x_2 = 0.530$, $x_4 = 0.0$.

The probability of system failure for the deterministic optimum was estimated using the procedure described in the appendix. For each mode, failure was defined to occur when the damping ratio of that mode was less than 2.5%. System failure was defined to occur when any of the damping ratios of the first four vibration modes fell below 2.5%.

Table 4 presents the results of a reliability analysis of the deterministic design. Mode 2 is the most critical mode, and mode 3 is the second most important mode. The probability of failure of mode 4 is small compared to those of the other modes. The safety of the design is indicated by the system failure probability, $P(F_s) = 4.81\%$. Another measure

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of the safety of the design – the system reliability index – is also given in Table 4. The system reliability index, denoted β_S, is defined by

$$R_S = \Phi(\beta_S) \tag{5}$$

in which $\Phi(.)$ is the probability distribution function of a standard Gaussian random variable, and R_S is the system reliability.

Table 4. Probabilities of Failure of Modes, System Probability of Failure, and System Reliability Index of Deterministic Design

Failure probability of mode 1 (%)	Failure probability of mode 2 (%)	Failure probability of mode 3 (%)	Failure probability of mode 4 (%)	System failure probability (%)	System reliability index
1.06	4.65	2.28	0.23	4.81	1.664

The system failure probability was also estimated using Monte-Carlo simulation with 10,000 samples. This probability was found to be 5.23%, which is approximately 9% larger than the probability estimated using the combination of second-moment methods and the second-order, upper Ditlevsen bound.

Regarding the relative importance of the modes to system safety, deterministic analysis and probabilistic analysis reach different conclusions. Specifically, deterministic analysis indicates that mode 3 is the least important of the four modes (Figure 3). However, a probabilistic analysis of this deterministic design indicates that mode 3 is the second most important mode. Indeed, according to Table 4, the probability that the damping ratio is less than 2.5% (the value that corresponds to failure) is more than twice as great in the third vibration mode as it is in the first vibration mode. Calculations show that the standard deviation of the limit state function of mode 3 is larger than that of mode 1. As a result, the mean value of the damping ratio is 2.3 standard deviations away from 2.5% for mode 1, and 2.0 standard deviations for mode 3, which means that mode 3 is more important than mode 1.

The logarithmic sensitivity derivatives[2] of the reliability indices for the 4 failure modes with respect to the standard deviations of the gains and damping factors are presented in Table 5. It is observed that:

- The reliability index for failure mode 1 is only sensitive to the uncertainties in the gains of the active members (members 1 and 2).

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[2] As used herein, the logarithmic sensitivity derivative of $f(x)$ w.r.t. x is given by $x\dfrac{d}{dx}(log_e f)$ which is $x\dfrac{f'}{f}$. This derivative gives the relative change in f caused by a unit relative change in x, or it can be interpreted as the percent change in f caused by a one percent change in x. If f is the reliability index and x_i are the standard deviations of the random variables, then the sum of the logarithmic derivatives is unity, as shown in Table 5.

- The reliability indexes of failure modes 2 and 3 are only sensitive to the gain of member 1 .

- Both active members and the passive damper (member 3) provide damping to vibration mode 4. However, the reliability index for failure in this mode is more sensitive to the uncertainties in the gain of the second active member and the passive damper than to the uncertainty in the gain of the first member.

- The second passive damper (member 4) is unimportant - none of the failure modes is sensitive to the damping factor of this member.

The effect of the normalized gain of active member 1 on the damping ratios of the four vibration modes is depicted in Figure 4. Modes 2 and 3 are the most sensitive while mode 4 is the least sensitive. The optimum design is at the crossing of the lines representing modes 1, 2, and 4.

Table 5. Deterministic Design: Logarithmic Sensitivity Derivatives of Reliability Indices for Failure Modes with Respect to Standard Deviations of Gains and Dampers

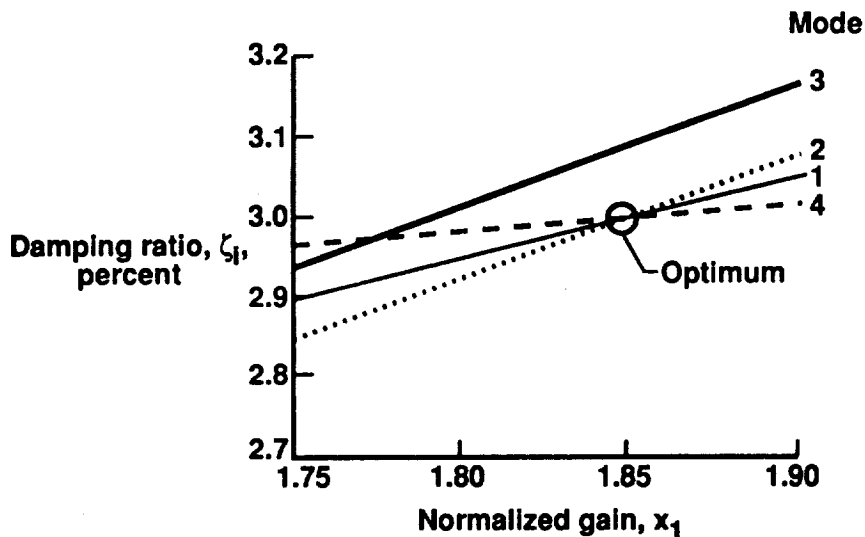| Mode \ Member number	Active members		Passive members	
	1	2	3	4
Failure Mode 1	0.89	0.11	0	0
Failure Mode 2	1.0	0	0	0
Failure Mode 3	1.0	0	0	0
Failure Mode 4	0.18	0.44	0.37	0

Figure 4.- Deterministic optimum. Effect of normalized gain x_1 on damping ratios of first four vibration modes.

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Probabilistic Design

Table 6 presents the probabilities of failure of the four modes and the system failure probability of the probabilistic design. The system reliability index of the probabilistic design was calculated to be 1.921, which corresponds to a probability of failure of about 2.7%. The failure probability was also found to be about 2.7% using Monte Carlo simulation. By comparing Table 6 with Table 4, it can be seen that the ranking of the failure modes is different. For example, the probabilistic design is more likely to fail under mode 1, while the deterministic design is more likely to fail under mode 2. It is also observed that the probabilities of failure of the modes of the deterministic design differ more than those of the probabilistic design.

Table 6. Probabilities of Failure of Modes, System Probability of Failure, and System Reliability Index of Reliability-Based Design

Failure probability of mode 1 (%)	Failure probability of mode 2 (%)	Failure probability of mode 3 (%)	Failure probability of mode 4 (%)	System failure probability (%)	System reliability index
2.2	2.0	1.0	0.23	2.739	1.921

Figure 5 shows how the probability of system failure and the cost vary with the mean values of both the gain, x_1, of active member 1 and the damping factor, x_3, of passive member 3. In this figure, the mean value of the gain, x_2, of active member 2 is equal to 0.413, which is the value that x_2 takes at the optimum. The solid curves correspond to constant values of the probability of system failure, $P(F_s)$. The dashed lines correspond to constant values of total cost, c. The reliability-based optimum is indicated by the triangular symbol.
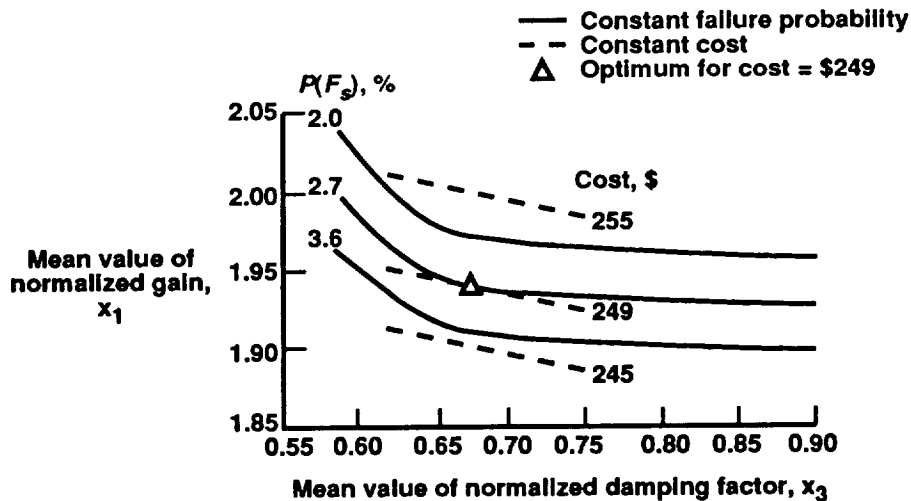

Figure 5.- Design space for reliability-based optimum. Curves define constant values of system failure probability and total cost. $x_2 = 0.413$, $x_4 = 0.0$.

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At the optimum, the line that corresponds to a cost of $249 is tangent to the curve that corresponds to a probability of failure of about 2.7%. Starting at the optimum, a move to the left or right along that $249 line causes the probability of system failure to increase. This confirms the fact that, out of all the designs that cost $249, the probabilistic optimum (triangular symbol) has the smallest probability of system failure.

Figure 6 shows how the safety of optimum probabilistic designs varies with the maximum allowable cost. The vertical scale at the left is the system reliability index. The vertical scale at the right is the probability of system failure. The horizontal scale is the maximum allowable cost. Over the range of costs shown, the safety of the system increases as the allowable cost increases. The triangular symbol indicates the failure probability (rather than the reliability index) of the deterministic optimum.
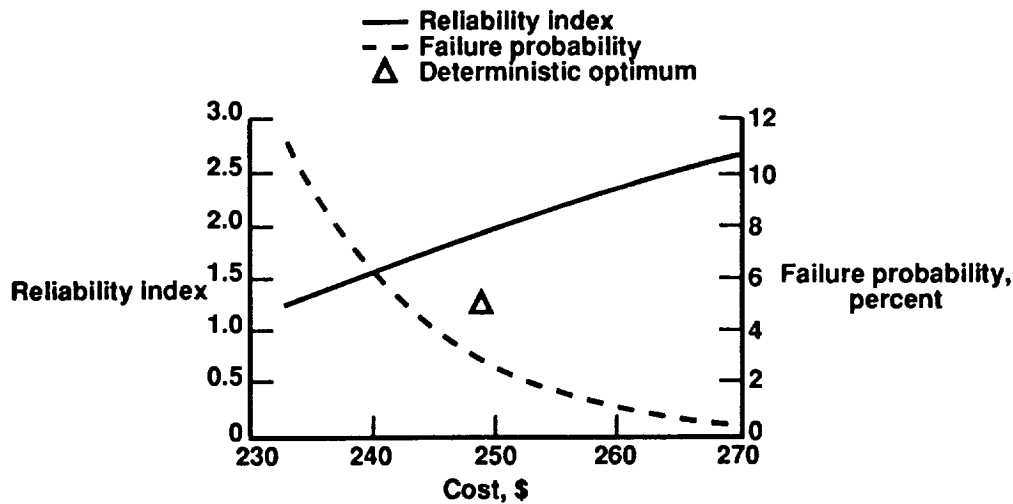
Figure 6.- System reliability index, β_S, and system probability of failure, $P(F_S)$, versus total cost for optimum probabilistic designs. Triangular symbol indicates failure probability of deterministic optimum.

Comparison

In the reliability-based optimization procedure used herein, the probability of system failure is minimized subject to the requirement that the cost is less than an upper limit. The cost is the sum of the costs of the active members and the passive dampers. Therefore, the optimality criterion for the probabilistic optimum is:

$$\frac{\partial P(F_s)}{\partial c_i} = constant \quad for \ i=1,...,3 \qquad (6)$$

where c_i is the cost of the ith member. According to this criterion, at the optimum, the three sensitivity derivatives in equation (6) are all equal. The three sensitivity derivatives for the deterministic and probabilistic optima are presented in Figure 7.

For the deterministic optimum, the sensitivity derivatives are not equal; therefore, the deterministic optimum violates the optimality criterion. The sensitivity derivative with respect to the cost of member 1 (the first active member) is largest, and the sensitivity derivative with respect to the cost of member 2 is the smallest. Therefore, by increasing the gain of member 1 and reducing the gain of the other active member and/or the damping of the passive member, the safety of the system can be improved without exceeding the budget.

In contrast, for the reliability-based optimum, the sensitivity derivatives are almost identical. This means that, as long as the total cost is fixed, the safety cannot be increased by changing the gains and the damping factor.
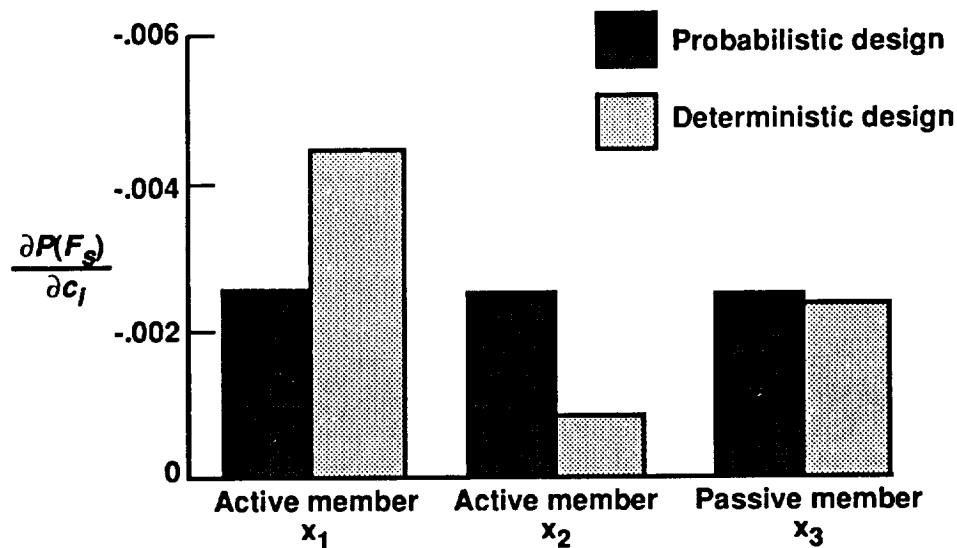
Figure 7.- Derivatives of system failure probability with respect to member cost for probabilistic and deterministic optima.

Table 7 presents the probabilistic optimum and compares it with the deterministic optimum. It is observed that, although the two optima cost the same, the probabilistic optimum is considerably safer than the deterministic optimum. Specifically, the probability of failure of the probabilistic optimum is about 56% of that of the deterministic optimum. Therefore, in an experiment, it is likely that the probabilistic design would perform better than the deterministic design.

Reliability-based optimization yielded a better design than deterministic optimization because it had the following advantages compared with deterministic optimization:

- It accounted for some of the uncertainties in a rational way.
- It accounted for the sensitivity of the cost and performance of the system with respect to these uncertainties.

The reliability indices of the two optima were also evaluated using Monte-Carlo simulation. The results, which are also presented in Table 7, agree with those obtained using the combination of second-moment methods and the second-order, upper Ditlevsen bound.

Table 7. Comparison of the Deterministic and Probabilistic Optima

	Deterministic optimum	Probabilistic optimum
x_1	1.8483	1.941
x_2	0.53	0.413
x_3	0.56	0.675
Cost ($)	249.0	249.0
$P(F_S)^1$ $(\beta_S)^1$	0.048 (1.664)	0.027 (1.921)
$P(F_S)^2$ $(\beta_S)^2$	0.052 (1.628)	0.027 (1.927)

[1] estimated using the second-order, upper Ditlevsen bound.
[2] estimated using Monte-Carlo simulation with 10,000 replications.

Conclusions

This report has proposed and described an analytical and experimental study for comparing reliability-based optimization with deterministic optimization. The study involves designing, testing, and comparing alternative structural designs obtained by probabilistic and deterministic methods. The experimental portion of the study is practical because of the following two important features:

- The study uses a modular truss that consists of many identical members. After the truss is tested, it can be disassembled and its members rearranged. Then it can be reassembled and retested.

- The designs are compared by testing their dynamic performance. Failure is defined to occur if the dynamic performance is unacceptable. Thus, failure does not imply destruction of the structure.

These two features allow many random samples of the same structure to be constructed and tested using a small number of members.

The following are the conclusions from an analytical example presented herein:

- In the example, reliability-based optimization produced a design that was more reliable than, and cost the same as, the corresponding deterministic optimum. Since reliability-based optimization accounts for more information than deterministic optimization, it can lead to a better design.

- Deterministic analysis incorrectly ranked the failure modes in terms of their importance to system reliability.

Acknowledgment

Appendix: Reliability Assessment Methodology

The procedure for reliability assessment of a system consists of two steps: element reliability analysis and system reliability analysis. In *element* reliability analysis, the analyst calculates the probability that each of the failure modes of the elements of a system may occur. In *system* reliability analysis, the analyst calculates the probability that the system may fail due to the failure of its modes.

In the example considered, there were four failure modes - one for each of the first four vibration modes. Each failure mode occurs if the damping ratio of the corresponding vibration mode becomes less than a specified minimum acceptable value. System failure occurs if any of the four damping ratios is less than the minimum acceptable value for that vibration mode. In general, the minimum acceptable value can depend upon the vibration mode. However, for the example considered, it was the same for each vibration mode, namely, 2.5%.

Element Reliability Analysis

The failure probabilities of the four failure modes were estimated using a second-moment algorithm (Madsen et al. [86]). The following is a brief description of that algorithm.

Let the performance function g_i for the ith failure mode of a general structural system be defined as

$$g_i(\mathbf{X}) = R_i(\mathbf{X}) - L_i(\mathbf{X}) \tag{7}$$

where $\mathbf{X}$ is the vector of random variables, $L_i(\mathbf{X})$ is the load effect (e.g., stress), and $R_i(\mathbf{X})$ is the resistance of the structure (e.g., allowable stress). Failure occurs when the load effect exceeds the resistance. The failure probability of the ith failure mode is the probability that the performance function corresponding to that mode becomes negative:

$$P(F_i) = P(g_i(\mathbf{X}) \leq 0) \tag{8}$$

If the random variables are Gaussian and the performance function is a linear function of the random variables, then there is an analytical, closed-form solution for the failure probability (Madsen et al. [86]). However, in our problem, the performance function is a nonlinear function of the random variables because the damping of a mode is a nonlinear function of both the gains of the active members and the damping factors of the passive

members. For that reason, an approximate method, denoted the second-moment algorithm, is used to calculate $P(F_i)$.

The key idea of the second-moment algorithm is to linearize the performance function in terms of the values of the random variables using a Taylor series expansion. The linearization point is that combination of values of random variables that has the highest probability to occur and makes the performance function zero. This point is called the *most probable failure point* or the *design point*. Once the performance function is linearized, it is straightforward to evaluate the failure probability.

To find the most probable failure point, the random variables X are first transformed into Gaussian, independent random variables, which have zero mean and unit standard deviation. These variables are called reduced random variables (denoted Z), and the space defined by these variables is called reduced space. In reduced space, the most probable failure point lies on the surface $g_i(Z) = 0$ and is closest to the origin. Optimization techniques are used to determine the most probable failure point, Z^*, (Madsen et al. [86], Liu and Der Kiureghian [86]). The distance from the origin to the most probable failure point is called the reliability index. It is related to the reliability and probability of failure by

$$R_i = 1 - P(F_i) = \Phi(\beta_i) \tag{9}$$

where β_i is the reliability index for the ith failure mode, $\Phi(.)$ is the cumulative probability distribution function of a standard Gaussian random variable, R_i is the reliability associated with the ith mode, and $P(F_i)$ is the probability of failure of the ith mode.

The optimization problem to be solved is defined by

Find Z to:

Minimize: $|Z|$

Subject to: $g_i(Z) = 0$ (10)

where Z is the vector of m random variables, $z_1,..., z_m$, in the reduced space. The solution to this optimization problem is the most probable failure point, Z^*. The values of the reduced random variables at the most probable failure point are $z_1^*,..., z_m^*$. The most probable failure point in the space of the original random variables is X^*. The corresponding values of the original random variables are $x_1^*,..., x_m^*$.

In the case of the ten-bar truss, there were three random variables: the gains of the two active members, x_1 and x_2, and the damping factor of the passive member x_3. There were four failure modes corresponding to the vibratory modes of the truss. The performance function of the ith failure mode was

$$g_i(X) = \zeta_i(X) - \zeta_{0,i} \qquad i=1,..,4 \tag{11}$$

Because the random variables were Gaussian and independent, the reduced random variables were obtained from the original random variables using the following transformation:

$$z_j = \frac{x_j - \mu_{x_j}}{\sigma_{x_j}} \qquad (12)$$

where x_j is the jth random variable, and μ_{x_j} and σ_{x_j} are its mean and standard deviation, respectively.

Tables 8 and 9 present the most probable failure points of the deterministic and probabilistic designs, respectively, for each failure mode.

Table 8. Deterministic optimum: Most probable failure points corresponding to the four failure modes.

Variable\Mode	1	2	3	4
$x_1{}^*$	1.447	1.538	1.479	1.623
$x_2{}^*$	0.488	0.529	0.526	0.430
$x_3{}^*$	0.558	0.559	0.558	0.462
$z_1{}^*$	-2.171	-1.680	-1.997	-1.218
$z_2{}^*$	-0.775	-0.024	-0.081	-1.888
$z_3{}^*$	-0.025	-0.004	-0.027	-1.732

Table 9. Probabilistic optimum: Most probable failure points corresponding to the four failure modes.

Variable\Mode	1	2	3	4
$x_1{}^*$	1.564	1.543	1.490	1.695
$x_2{}^*$	0.392	0.4123	0.410	0.353
$x_3{}^*$	0.673	0.6746	0.673	0.535
$z_1{}^*$	-1.947	-2.054	-2.325	-1.269
$z_2{}^*$	-0.515	-0.022	-0.070	-1.455
$z_3{}^*$	-0.025	-0.005	-0.037	-2.068

System Reliability Analysis

In system reliability analysis, the failure probabilities of the individual modes (obtained from element reliability analysis) are combined to evaluate the failure probability of the system. Systems that fail if any of the individual failure modes occurs are called series systems. In this study, it is assumed that the truss fails if any of the damping ratios of the first four modes is less than a minimum acceptable value. Therefore, the truss is a series system.

The probability of system failure is

$$P(F_s) = P(\bigcup_1^n F_i) \tag{13}$$

where

F_s is the event of failure of the system,

F_i is the ith failure event (failure mode),

$\bigcup_1^n F_i$ is the union of the failure modes,

$P(F_s)$ is the probability of failure of the system, and

n is the number of failure modes, which, in this example, is equal to the number of vibratory modes, which is 4.

The equation for the system failure probability involves the joint probabilities of occurrence of all combinations of failure modes and is given by

$$P(F_s) = \sum_{i=1}^{4} P(F_i) - \sum_{\substack{i=1 \\ i<j}}^{3} \sum_{j=2}^{4} P(F_i F_j) + \sum_{\substack{i=1 \\ i<j<k}}^{2} \sum_{j=2}^{3} \sum_{k=3}^{4} P(F_i F_j F_k) - P(F_1 F_2 F_3 F_4) \tag{14}$$

where, for example, $P(F_i F_j)$ is the joint probability of occurrence of modes i and j. It is difficult to calculate the joint probabilities of the modes because the boundaries of the failure regions are not known explicitly and because the calculation requires multiple nested integrations.

The second-order, upper Ditlevsen bound approximates the system failure probability using only the probabilities of the individual modes and the joint probability of occurrence of the combinations of modes taken two at a time (Ditlevsen [79]). The bound is given by

$$P(F_s) \cong \sum_{i=1}^{n} P(F_i) - \sum_{\substack{i=2 \\ j<i}}^{n} max P(F_j F_i) \tag{15}$$

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where n is the number of modes, which, in this study, is 4. In the second term on the right in equation (15), a maximum value of $P(F_j F_i)$ is selected for each value of i; the subscript j ranges from 1 to i-1. The joint probabilities in equation (15) were evaluated by using the linearized performance functions determined in element reliability analysis. That linearization provided explicit expressions for the failure boundaries. The approximation given by equation (15) was used herein and provided estimates of the system failure probability given in Table 7.

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An analytical and experimental study for assessing the potential of reliability-based structural optimization is proposed and described. In the study, competing designs obtained by deterministic and reliability-based optimization are compared. The experimental portion of the study is practical because the structure selected is a modular, actively and passively controlled truss that consists of many identical members, and because the competing designs are compared in terms of their dynamic performance and are not destroyed if failure occurs. The analytical portion of this study is illustrated on a 10-bar truss example. In the illustrative example, it is shown that reliability-based optimization can yield a design that is superior to an alternative design obtained by deterministic optimization. These analytical results provide motivation for the proposed study, which is underway.

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